Enhanced relativistic High-Harmonic Generation (HHG) using tailored pulses

MICHAEL KLAIBER[1,2], KAREN Z. HATSAGORTSYAN[1], AND CHRISTOPH H. KEITEL[1]

[1]*Max-Planck-Institut für Kernphysik Heidelberg, Germany*

[2]*Theoretische Quantendynamik Universität Freiburg, Germany*

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1. Problem

The HHG process in an atomic system:

- Ionization of a bound electron
- Propagation of the ionized electron in the continuum → energy gain
- Recombination of the ionized electron with the atomic core → Emission of high-harmonics

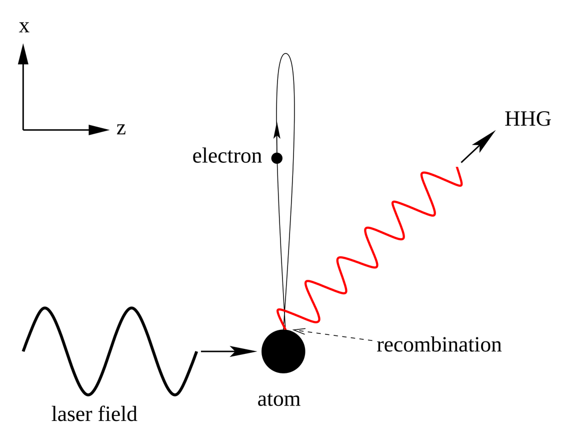
→ **the HHG process is a potential source of highly energetic radiation**

Parameters:

- optical laser wavelength $\lambda = 456$ nm (angular frequency $\omega = 0.1$ a.u.)
- atomic ionization potential I_p at the borderline between tunneling and over-the-barrier ionization
- laser intensities up to 5×10^{19} W/cm^2, i.e. strongly relativistic

The nonrelativistic regime [1]:

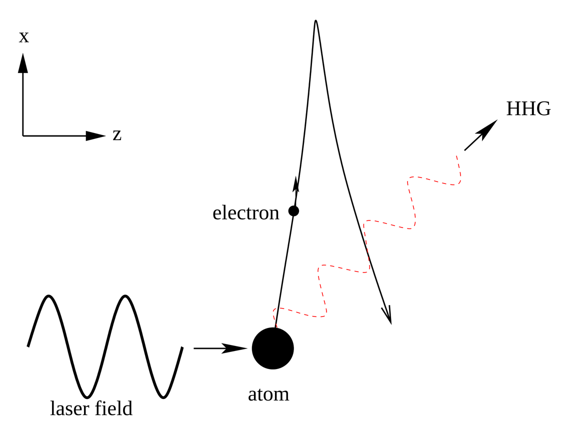
- $v/c \ll 1$, laser magnetic field neglectable, no drift of the ionized electron in laser propagation direction
- effective recombination of the ionized electron with the atomic core → high emission rate of HHG
- nonrelativistic description valid up to intensities of 10^{17} W/cm^2
- maximal energy of the emitted radiation: $3U_p + I_p = 6$ keV



The relativistic regime [2]:

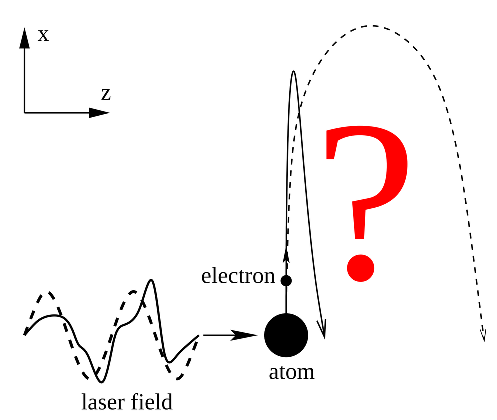
- laser magnetic field nonnegligible, drift of the ionized electron in laser propagation direction → most of the electron wave packet misses the atomic core
- emission rate strongly suppressed

→ **It is not possible to generate highly energetic radiation with the HHG process in an atomic system by a simple increase of the laser intensity**



2. Idea

Is it possible to reduce the drift of the ionized electron significantly by changing the shape of the laser field from sinusoidal oscillations to specially tailored laser pulses?



3. Theory

- neglecting the spin of the ionized electron its dynamics is described by the Klein-Gordon equation
- applying the single-active electron approximation

$$\left\{\left(i\partial^\mu + A^\mu/c + A^\mu_H/c + g^{\mu 0}V/c\right)^2 - c^2\right\}\Psi(x) = 0$$

Amplitude of the HHG process within the strong-field approximation (SFA) [3]:

$$M_n = -i\int d^4x' \int d^4x'' \,\Phi(x')^* V_H(x') G^V(x',x'') V_A(x'')\Phi(x'')$$
$$= \int_{-\infty}^{\infty} d\eta' \int_{-\infty}^{\eta'} d\eta'' \int d^3\mathbf{q}\, m^H(\mathbf{q},\eta',\eta'') \exp\left\{-i\left[S(\mathbf{q},\eta',\eta'') - n\eta'\right]\right\}$$

- $V_H(x) = 2\mathbf{A}_H(x)\cdot(\hat{\mathbf{p}} + \mathbf{A}(\eta)/c)$ part of the electron-field interaction operator
- $V_A(x) = 2iV/c^2\partial_t + V^2/c^2$ electron-ion interaction operator
- $S(\mathbf{p},\eta,\eta') = \int_{\eta'}^{\eta} d\tilde\eta \left(\varepsilon_{\mathbf{p}}(\tilde\eta) - c^2 + I_p\right)/\omega$ the quasiclassical action

Differential rate of HHG with emission angle Ω:

$$\frac{dw_n}{d\Omega} = \frac{\omega^2}{(2\pi)^3}\frac{n\omega}{c^3}|M_n|^2$$

Saddle-point method: $\partial_i(S(q,\eta',\eta'') - n\eta') = 0$

- valid in the long wavelength regime: $\omega < I_p$
- defines the recombination phase $\Re(\eta')$, the ionization phase $\Re(\eta'')$ and the drift momentum $\tilde{\mathbf{q}}$ of the ionized electron
- $\Im(\eta'')$ corresponds to the ionization probability

4. The optimized tailored pulse

Construction of the optimized pulse [4]:

- Analysis of the saddle points with the Fourier expansion of an arbitrary pulse $E(\eta) = \sum_{k=-K}^{K} c_k \exp(ik\omega_0\eta)$
- Variation of the coefficients c_k
- Minimizing $\Im(\eta'')$

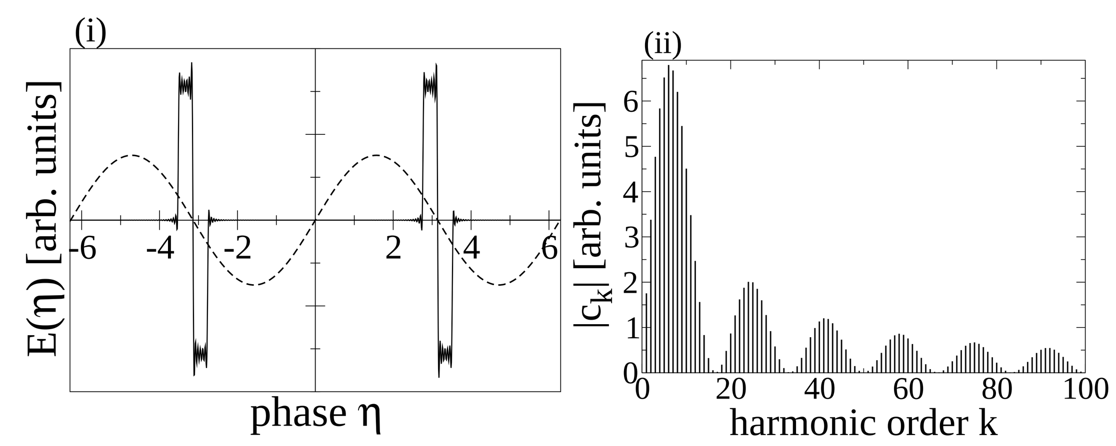
→ optimized pulse

Features of the optimized pulse:

- strong decrease of the laser field strength after the phase of ionization
  → concentration of the ionization force on a short time interval
- long time span with weak laser field strength after ionization
  → Propagation with nonrelativistic velocity, i.e. no drift
- strong and short pulse shortly before recombination
  → concentration of the acceleration force on a short time interval, i.e. fast energy gain, little drift

The optimized pulse:

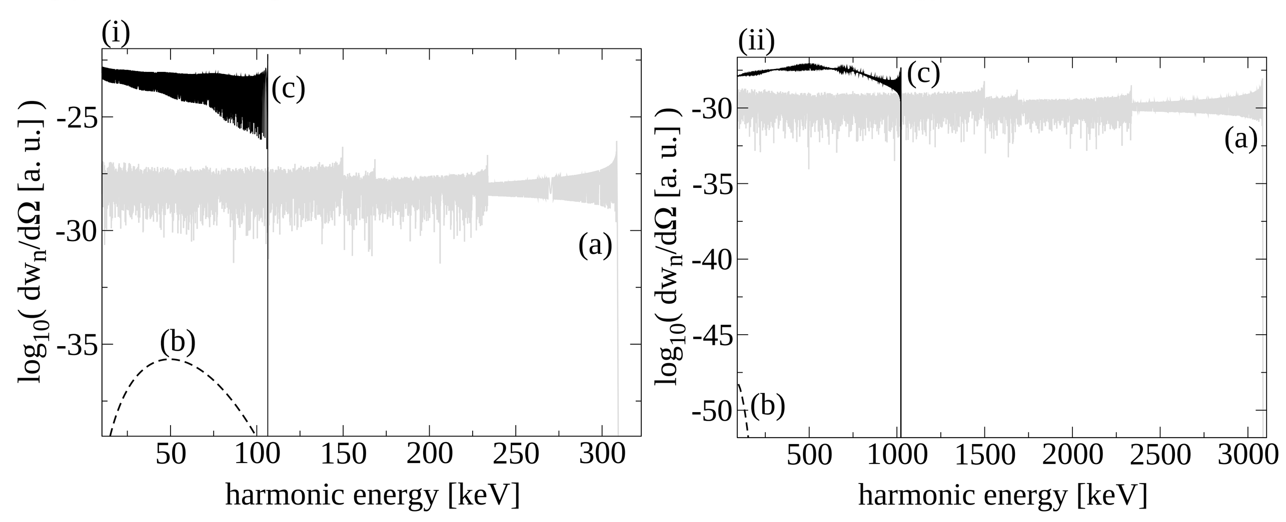
- Synopsis of the discovered features plus simplification
  → attosecond pulse train with rectangular pulses with a duration of 90 as (laser period 1.5 fs)



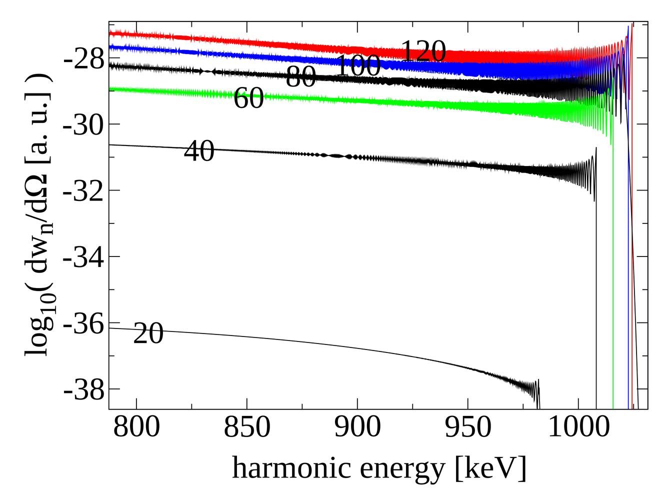
(i) Temporal dependence of the tailored pulse compared with a sinus wave with the same averaged intensity.
(ii) The frequency spectrum of the tailored pulse. The phase spectrum of the tailored pulse is linear, i.e. the phases are locked.

5. Spectra

(i) moderately relativistic regime → **maximal harmonic emission energy of 100 keV**
(ii) strongly relativistic regime → **maximal harmonic emission energy of 1 MeV**



Harmonic emission rate in laser polarization direction via $\log_{10}(dw_n/d\Omega)$, as function of the harmonic energy with a laser intensity of (i) 5×10^{18} W/cm^2 and (ii) 5×10^{19} W/cm^2. The main angular frequency is $\omega = 0.1$ a.u.. The ionization potential is (i) $I_p = 28$ a.u. and (ii) $I_p = 62$ a.u., respectively: (a, grey) within the dipole approximation and a sinusoidal field, (b, dashed) with respect to the Klein-Gordon equation and a sinusoidal field, (c, black) with respect to the Klein-Gordon equation and a tailored pulse.



Harmonic emission spectra in laser polarization direction via $\log_{10}(dw_n/d\Omega)$, as function of the harmonic energy with a laser intensity of 5×10^{19} W/cm^2 using the tailored pulse, which consists of 20, 40, 60, 80, 100, and 120 harmonics of the main angular frequency. The ionization potential is $I_p = 62$ a.u..

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[4] M. Klaiber, K.Z. Hatsagortsyan, and C.H. Keitel, to be published (2006).